

8. Find equation of plane through line :

$$\frac{x-1}{3} = \frac{y+6}{4} = \frac{z+1}{2}$$

and parallel to :

$$\frac{x-2}{2} = \frac{y-1}{-3} = \frac{z+4}{5}$$

9. Find the volume of tetrahedron bounded by coordinate planes and the plane $x + y + z = 1$.

Roll No. 0102685

BCA-205(N)

B. C. A. (Second Semester)
EXAMINATION, May, 2019

(New Course)

Paper Fifth

MATHEMATICS-II

Time : Three Hours]

[Maximum Marks : 75

Note : Attempt questions from all Sections as directed.

Inst. : The candidates are required to answer only in serial order. If there are many parts of a question, answer them in continuation.

Section—A

(Short Answer Type Questions)

Note : Attempt all questions. Each question carries 3 marks.

1. (A) Prove that for any two non-empty sets :

$$A' - B' = B - A.$$

(B) Let $A = \{1, 2, 3\}$ and relation R on A be defined by $R = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$.

Find R^{-1} .

[2]

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- (C) Draw Hasse diagram of $(A, /)$, where $A = \{1, 2, 3, 4, 6, 9\}$ and $'/'$ is divisibility relation.

- (D) If:

$$u = \frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}},$$

find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$.

- (E) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ when $z = \ln(x^2 + y^2)$.

- (F) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that:

$$f(x) = \sqrt{x^2 + 1}$$

and $g: \mathbb{R} \rightarrow \mathbb{R}$ be such that $g(x) = x$. Find $f \circ g$ and $g \circ f$. Here $\mathbb{R}$ is set of all real numbers.

- (G) Show that the points $(3, -2, 4)$; $(1, 1, 1)$ and $(-1, 4, -2)$ are collinear.

- (H) Find direction cosines of line segment joining $P(2, 3, 6)$ and $Q(3, -4, 5)$.

- (I) Evaluate:

$$\int_1^2 \int_0^x \frac{dx dy}{x^2 + y^2}$$

Section—B

(Long Answer Type Questions)

Note: Attempt any two questions. Each question carries 12 marks.

2. Prove that the relation " $x \equiv y \pmod{m}$ " given by $R = \{(x, y) \mid x - y \text{ is divisible by } m\}$ is an equivalence relation.

(B-11)

[3]

BCA-205(N)

3. Let L be set of all factors of 12 and let $'/'$ be divisibility relation on L . Show that $(L, /)$ is lattice.

4. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x) = 2x - 3$

and $g: \mathbb{R} \rightarrow \mathbb{R}$ be such that $g(x) = \frac{x-3}{2}$. Find:

(i) $g \circ f$

(ii) f^{-1}

(iii) $(g \circ f)^{-1}$

5. (a) Prove that:

$$(A \cap B)' = A' \cup B'$$

- (b) If $A = \{1, 4\}$, $B = \{4, 5\}$ and $C = \{5, 7\}$ verify that:

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

Section—C

(Long Answer Type Questions)

Note: Attempt any two questions. Each question carries 12 marks.

6. If $u = \tan^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$, prove that:

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4 \sin^2 u) \sin 2u$$

7. Discuss maxima and minima of $f(x, y) = xy(1 - x - y)$.

(B-11) P. T. O.